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Phase Dynamics of a Closed 0-π Josephson Junction

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Abstract. The phase dynamics of a closed 0-π junction is investigated. Such a system can be realized, for example, in a small d(x^2-y^2)-wave superconductor embedded in a conventional s-wave superconducting matrix. The Hamiltonian of the system is derived. We prove that the system can be mapped on a quantum two-level system when the system size is small enough.

Keywords: pi-junction, two-level system, Josephson effect.
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INTRODUCTION

In 0-pi Josephson junction systems, which includes both the regions of $j_c > 0$ and $j_c < 0$, j_c being the Josephson critical current density, vortices (a vortex) with fractional flux quantum spontaneously appear(s) [1-3]. Such 0-pi junction systems can be fabricated, using d(x^2-y^2)-wave and s-wave superconductors. In this paper we theoretically investigate the phase dynamics of a closed 0-pi Josephson junction system in which a small square-shape d(x^2-y^2)-wave superconductor is embedded in a s-wave superconducting matrix. We show that the closed 0-pi junction system can be considered as a quantum two-level system if the system size is small enough.

FORMULATION

We study phenomenologically the Josephson phase dynamics in a small square-shape d(x^2-y^2)-wave superconductor (d-dot) embedded in a conventional s-wave superconductor. The crystallographic orientation of the d-dot is shown in Fig.1. Suppose that the interface between the d(x^2-y^2)-wave and s-wave superconductors form the S-I-S Josephson junction. Since in this system the Josephson critical current density j_c changes its sign at the four corners of the square d-dot, this system forms a closed 0-π junction system. It is well known that a vortex with half-flux quantum spontaneously appears in 0-π junctions if the system size is large enough [1- 3]. Let the length of a side of the d-dot be a which is assumed much larger than the in-plane London penetration depths of both d- and s-wave superconductors. In this case the overlap of the magnetic flux on the different sides of the d-dot may be neglected. Hence, the system can be mapped on a one-dimensional Josephson junction model with the boundary condition,

$$\theta(x+4a,t)=\theta(x,t)+\frac{2\pi m}{4a}x. \qquad (1)$$

Where $\theta(x,t)$ is the phase difference at position x ($0<x<4a$) and at time t and m is an integer. Thus, one may assume that the under-damped phase dynamics in this system is described by the generalized sine-Gordon equation as

$$\frac{\varepsilon}{c^2}\partial_t^2\theta(x,t)-\partial_x^2\theta(x,t)=\frac{\eta(x)}{\lambda_J^2}\sin\theta(x,t), \qquad (2)$$

where λ_J is the Josephson penetration depth and $\eta(x)$ is a function taking either of two values, 1 or -1, depending on the sign of j_c, i.e., $\eta(x)=1$ for the region of $j_c > 0$ and $\eta(x)= -1$ for. $j_c < 0$. Let the thickness of the insulating barrier and the width of the junction along the [001]direction be D and L respectively. In this case the Hamiltonian that leads to Eq.(2) is obtained as,

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$$H = E_J \int_0^{4a} dx\{\frac{c^2}{2\varepsilon E_J^2}\Pi(x)^2 + \frac{1}{2}(\partial_x\theta(x))^2 \tag{3}$$
$$-\frac{1}{\lambda_J^2}\eta(x)\cos\theta(x)\},$$

where $\Pi(x)$ is the canonical momentum of $\theta(x)$ and $E_J=(L/4\pi D)/(\phi_0/2\pi)$ with ϕ_0 being the unit flux. In the following we investigate the case of $\lambda_L< a << \lambda_J$, which may be realized, for example, when a ~ 1μm In this case the spatial variation of the phase difference is expected to be very weak, since λ_J gives the spatial scale, as seen in Eqs.(2) and (3). In order to describe this situation it is convenient to introduce the Fourier series expansions as

$$\theta(x) = Q + \frac{m\pi}{2a}x + \sqrt{2}\sum_{n=1} q_n \sin\frac{\pi n}{2a}x, \tag{4}$$

$$\Pi(x) = P/4a + (1/2\sqrt{2})\sum_{n=1} p_n \sin\frac{\pi n}{2a}x. \tag{5}$$

In Eqs.(4) and (5) we retained only the sine-components, because the cosines do not appear as a result of the symmetry of the system. In this paper we restrict ourselves to the case of m=0, i.e., the zero external field case. Note that if the canonical commutation relations are imposed for the Fourier coefficients,

$$[Q,P] = i\hbar, \quad [q_n, p_m] = i\hbar\delta_{nm},$$
$$[Q,p_n] = [q_n, P] = 0, \tag{6}$$

one may construct a quantum theory for the phase dynamics of the 0-π junction system. Since the higher harmonics components (q_n,p_n) are small in the present small d-dot system, the Josephson coupling term in the Hamiltonian is approximated as

$$\cos\theta(x) \approx \cos Q$$
$$-\sqrt{2}\sin Q\sum_{n=1} q_n \sin\frac{\pi n}{2a}x + ..., \tag{7}$$

Then, substituting Eqs.(4) and (5) into Eq.(3) and using the approximation (7), one can express the Hamiltonian in terms of the Fourier components. Since the dynamics of the higher harmonics components are linear in this approximation, the variables (q_n, p_n) can be integrated out. Then, after some calculations one can obtain the Hamiltonian in terms of only the uniform component (Q,P) which is valid for the system of $a << \lambda_J$ as

$$H = \frac{P^2}{8Ma_J} - \frac{a_J^4}{6}\sin^2 Q, \tag{8}$$

where $M = (2\pi)^2\varepsilon E_J/h^2c^2$ and $a_J=a/\lambda_J$. Note that the minima of the potential appear at sin Q = 1 and -1, i.e., $Q = \pi/2$ and $-\pi/2$, in the region, $-\pi < Q < \pi$. These minima are the current-carrying states and the direction of the current at $Q = \pi/2$ is opposite to that at $Q = -\pi/2$. Thus, one concludes that the small d-dot system can be mapped on a quantum two-level system. When the quantum mechanical tunneling occurs between these two potential minimum states, the energy difference between the bonding and anti-bonding states is estimated from the Hamiltonian (8) as ΔE = 200 GHz for $L/4\pi D$ ~ 5, a_J ~ 0.1, λ_J ~ 0.001cm. The details of the derivation and analysis of Eq.(8) will be published elsewhere.

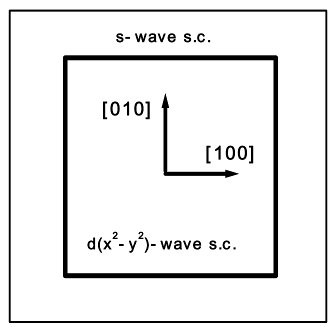

FIGURE 1. Schematic view of the d-dot system. The sign of the Josephson critical current changes at the corners.

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